MODELING THE SOLAR WIND USING A MAGNETOHYDRODYNAMIC EXTENSION OF PARKER'S MODEL

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Abstract

We present a mathematical model and numerical analysis of the solar wind, incorporating magnetohydrodynamic (MHD) effects. Extending Parker's classical solar wind model, we introduce a magnetic field through the dimensionless plasma beta parameter (β), defined as the ratio of thermal to magnetic pressure. The resulting model yields a nonlinear ordinary differential equation that governs the solar wind velocity profile as a function of radial distance from the Sun. Numerical solutions are obtained for a range of βvalues, revealing the influence of magnetic pressure on solar wind acceleration. Our analysis shows that lower values of β(indicating stronger magnetic effects) lead to earlier and more rapid acceleration through the critical point, producing higher terminal velocities. These findings underscore the importance of MHD processes in accurately describing solar wind dynamics and provide a theoretical framework for investigating solar-terrestrial interactions.

Keywords: Solar wind, Parker's model, Magnetohydrodynamics (MHD), Plasma beta (β), Critical point, Supersonic flow, Magnetic field effects, Space plasma physics

1. Introduction

The solar wind is a continuous outflow of charged particles, primarily electrons and protons, ejected from the solar corona into interplanetary space. First proposed theoretically by Parker (1958), the solar wind concept is fundamental to understanding space weather, planetary magnetospheres, and heliophysics. As the solar wind escapes the Sun's gravitational pull, it interacts with magnetic fields and structures throughout the solar system, forming phenomena such as the Parker spiral, the heliospheric current sheet, and geomagnetic storms near Earth.

The solar wind behaves as a plasma highly ionized gas responding to both fluid and electromagnetic forces. This makes magnetohydrodynamics (MHD), which combines fluid dynamics with Maxwell's equations, an ideal framework for modeling solar wind behavior. MHD allows for a self-consistent description of plasma flow and its interaction with solar and interplanetary magnetic fields (Verscharen et al., 2019).

A key result of Parker's original model is that the solar wind transitions from subsonic speeds near the Sun to supersonic speeds at larger distances, driven by thermal pressure overcoming solar gravity. The model predicts a critical point at which this transition occurs. The critical solution smoothly connects the inner solar corona to the outer heliosphere, providing fundamental insight into solar wind expansion (Cranmer, 2020).

Modern satellite missions such as NASA's Parker Solar Probe and ESA's Solar Orbiter provide in-situ measurements of solar wind properties, allowing theoretical models to be tested and refined (Fox et al., 2016; Müller et al., 2020). Understanding solar wind dynamics is essential not only for scientific research but also for space weather forecasting, satellite protection, and maintaining reliable terrestrial communication systems.

Parker's (1958) isothermal model solved the fluid equations under solar gravity and showed that plasma can continuously outflow and naturally reach supersonic speeds beyond a critical radius. Subsequent studies, including Weber and Davis (1967), incorporated magnetic fields, demonstrating that the solar magnetic field is "frozen in" to the plasma and produces the Parker spiral. Recent studies have extended the model to include non-isothermal effects, turbulence, and wave-particle interactions (Verscharen et al., 2019; Matthaeus et al., 2020). Observational data from missions such as Wind, ACE, Ulysses, and the Parker Solar Probe confirm key MHD predictions, including supersonic flow, spiral magnetic fields, and dynamic structures such as shocks and coronal mass ejections (Cranmer, 2020).

This study aims to revisit Parker's model mathematically, incorporate simplified MHD effects, and numerically investigate how magnetic fields and thermal pressure gradients influence solar wind velocity profiles and critical points.

2. Materials and Methods

2.1 Mathematical Formulation of Parker's Solar Wind Model

Assumptions of Parker's Model:

Parker (1958) formulated the solar wind model under the following simplifying assumptions:

1. Spherically symmetric, steady-state flow ($\partial/\partial t = 0$, independent of angular position).
2. Isothermal plasma: constant temperature T.
3. Single fluid approximation: the solar wind behaves as a compressible fluid of protons and electrons.
4. Neglect of magnetic and viscous forces: only gravity and pressure gradients are considered.
5. No energy generation or radiative losses: thermal energy is assumed to be constant.

Governing Equations:

Let $u(r)$be the radial velocity, $\rho(r)$the mass density, $p(r)$the pressure, $G$the gravitational constant, and $M$the solar mass.

Continuity Equation:

$\frac{1}{r^2}\frac{d}{dr}(r^2\rho u) = 0 \Rightarrow \rho u r^2 = \text{constant}$ (1)

This expresses mass flux conservation through a spherical shell.

Momentum Equation (radial component, neglecting viscosity and magnetic effects):

$\rho u\frac{du}{dr} = -\frac{dp}{dr} - \rho\frac{GM}{r^2}$ (2)

Isothermal Equation of State:

$p = \rho c_s^2, c_s = \sqrt{\frac{k_B T}{m}}$ (3)

where $c_s$is the constant sound speed, $k_B$is Boltzmann's constant, and $m$is the mean particle mass.

Derivation of the Parker Wind Equation:

From the momentum equation and isothermal condition:

$\rho u\frac{du}{dr} = -c_s^2\frac{d\rho}{dr} - \rho\frac{GM}{r^2}$ (4)

Using the continuity equation to eliminate $d\rho/dr$:

$\frac{d\rho}{dr} = -\rho(\frac{1}{u}\frac{du}{dr} + \frac{2}{r})$ (5)

Substituting (5) into (4) and simplifying:

$(u^2 - c_s^2)\frac{du}{dr} = \frac{2c_s^2 u}{r} - \frac{GMu}{r^2}$ (6)

Or equivalently:

$(u - \frac{c_s^2}{u})\frac{du}{dr} = \frac{2c_s^2}{r} - \frac{GM}{r^2}$ (Parker Wind Equation) (7)

Critical Point:

The flow passes smoothly through the critical point at:

$u = c_s, r_c = \frac{GM}{2c_s^2}$

Where $r_c$is the critical radius. The solution must pass through this point smoothly, which provides a uniquely, physically realistic solution.

Non-Dimensional Form of Parker's Wind Equation

Let us define the **normalized variables**:

$v = \frac{u}{c_s}$(normalized velocity), $x = \frac{r}{r_c}$(normalized radial distance)

We aim to express Parker's wind equation:

$(u - \frac{c_s^2}{u})\frac{du}{dr} = \frac{2c_s^2}{r} - \frac{GM}{r^2}$ in terms of $v$and x.

Step 1: Left-Hand Side (LHS)

$$\text{LHS} = (u - \frac{c_s^2}{u})\frac{du}{dr}$$
$$= (c_s v - \frac{c_s^2}{c_s v})\frac{d}{dr}(c_s v)$$
$$= c_s(v - \frac{1}{v})\cdot c_s\frac{dv}{dr}$$
$$= c_s^2(v - \frac{1}{v})\frac{dv}{dr}$$

Step 2: Right-Hand Side (RHS)

$\text{RHS} = \frac{2c_s^2}{r} - \frac{GM}{r^2} = \frac{2c_s^2}{r_c x} - \frac{GM}{r_c^2 x^2}$

Using the critical radius definition:

$r_c = \frac{GM}{2c_s^2} \Rightarrow GM = 2c_s^2 r_c$

Substitute into RHS:

$\frac{2c_s^2}{r_c x} - \frac{2c_s^2 r_c}{r_c^2 x^2} = \frac{2c_s^2}{r_c}(\frac{1}{x} - \frac{1}{x^2})$

Step 3: Combine LHS and RHS

$c_s^2(v - \frac{1}{v})\frac{dv}{dr} = \frac{2c_s^2}{r_c}(\frac{1}{x} - \frac{1}{x^2})$

Divide both sides by c_s^2:

$(v - \frac{1}{v})\frac{dv}{dr} = \frac{2}{r_c}(\frac{1}{x} - \frac{1}{x^2})$

Step 4: Change of Variable from $r$to x

Since $r = r_c x$, we have $dr = r_c dx$and thus:

$\frac{dv}{dr} = \frac{dv}{dx}\frac{dx}{dr} = \frac{1}{r_c}\frac{dv}{dx}$

Substitute into the previous equation:

$(v - \frac{1}{v})\frac{1}{r_c}\frac{dv}{dx} = \frac{2}{r_c}(\frac{1}{x} - \frac{1}{x^2})$

Multiply both sides by r_c:

$(v - \frac{1}{v})\frac{dv}{dx} = 2(\frac{1}{x} - \frac{1}{x^2})$ (8)

Equation (8) is the non-dimensional form of Parker's wind equation, suitable for numerical integration.

2.2 MHD Extension of Parker's Solar Wind Model

To incorporate magnetohydrodynamics (MHD) into Parker's solar wind model, we account for the effect of the interplanetary magnetic field (IMF) on solar wind dynamics. This extension provides a more physically comprehensive description of the solar wind, capturing the additional forces and acceleration due to magnetic fields.

2.2.1 Motivation for Including Magnetic Fields

The solar corona is magnetized, and as plasma escapes the Sun, it drags the magnetic field into interplanetary space. The plasma–magnetic field interaction is governed by the ideal MHD equations, where magnetic fields introduce **Lorentz forces** that can significantly alter the wind structure, including its acceleration and angular momentum transport.

2.2.2 Assumptions for MHD Extension

In addition to the previous assumptions of Parker's classical model, we now assume:

1. A steady, axisymmetric, and spherically expanding flow.
2. A magnetic field with radial and azimuthal components: $\mathbf{B} = B_r\hat{r} + B_\phi\hat{\phi}$.
3. Inclusion of magnetic pressure and tension (Lorentz force) in the momentum equation.
4. Perfect conductivity (ideal MHD), implying magnetic field lines are frozen into the plasma.

2.2.3 MHD Governing Equations

The steady-state radial momentum equation, including the azimuthal magnetic field, is:

$\rho u\frac{du}{dr} = -\frac{dp}{dr} - \rho\frac{GM}{r^2} + \frac{1}{4\pi}[B_\phi\frac{dB_\phi}{dr} + \frac{B_\phi^2}{r}]$ (9)

Here, the two magnetic terms represent:

Magnetic tension: $B_\phi\frac{dB_\phi}{dr}$ and Magnetic pressure: $\frac{B_\phi^2}{r}$

The solar magnetic field at the surface has a dipole structure, but due to solar rotation and the radial outflow, it is wound into a spiral (the Parker spiral). In a simplified form:

$B_r(r) = B_{r0}(\frac{R_\odot}{r})^2, B_\phi(r) = -B_r(r)\frac{\Omega r\sin\theta}{u(r)}$

where Ω is the solar angular velocity, and θ is the polar angle (often set to 90°for equatorial flow).

2.2.4 Modified Parker Equation with MHD

Substituting $B_\phi$into Equation (9) yields:

$\frac{B_\phi^2}{r} = \frac{B_r^2\Omega^2 r^2 \sin^2\theta}{u^2}, \frac{dB_\phi}{dr} = -[\frac{dB_r}{dr}\frac{\Omega r \sin\theta}{u} + B_r\frac{d}{dr}(\frac{\Omega r \sin\theta}{u})]$

Expanding the derivative:

$B_r\frac{d}{dr}(\frac{\Omega r\sin\theta}{u}) = B_r\Omega\sin\theta(\frac{1}{u} - \frac{r}{u^2}\frac{du}{dr})$

Then:

$B_\phi\frac{dB_\phi}{dr} = \frac{B_r^2\Omega^2 r^2\sin^2\theta}{u^2}\frac{dB_r}{dr} + \frac{B_r^2\Omega^2 r\sin^2\theta}{u^2} - \frac{B_r^2\Omega^2 r^2\sin^2\theta}{u^3}\frac{du}{dr}$

Finally, the MHD-modified momentum equation becomes:

$$(\rho u + \frac{B_r^2\Omega^2 r^2\sin^2\theta}{4\pi u^3})\frac{du}{dr} = -\frac{dp}{dr} - \rho\frac{GM}{r^2} + \frac{\Omega^2\sin^2\theta}{4\pi u^2}[B_r r^2\frac{dB_r}{dr} + 2rB_r^2] \quad (10)$$

where:

$u(r)$is the radial solar wind velocity; $B_r(r) = B_{r0}(R_\odot/r)^2$is the radial magnetic field

$\Omega$is the solar rotation rate; $\theta = 90°$for equatorial flow; $\rho$is the plasma density; $p = \rho c_s^2$for an isothermal plasma

Equation (10) explicitly shows that the azimuthal magnetic field contributes pressure-like and tension-like terms, enhancing solar wind acceleration, particularly in the outer corona.

2.2.5 Physical Implications

1. **Centrifugal magnetic force:** The azimuthal field exerts a centrifugal acceleration on the plasma, enhancing solar wind speed.
2. **Shifted critical point:** The flow reaches the modified magnetosonic speed at a new critical radius, rather than the pure sound speed.
3. **Angular momentum transport:** The solar wind carries angular momentum via magnetic torque, explaining long-term slowing of solar rotation (magnetic braking).
4. **Latitudinal dependence:** Acceleration profiles vary with latitude due to the $\sin^2\theta$factor, making MHD wind inherently anisotropic.

2.2.6 Comparison with Classical Parker Model

Feature	Parker Model (Hydrodynamic)	Parker Model (MHD)
Driving Mechanism	Thermal pressure gradient	Thermal + Magnetic forces
Critical Speed	Sound speed c_s	Magnetosonic speed
Magnetic Field	Neglected	Included (B_r, B_ϕ)
Angular Momentum	Not conserved	Magnetic torque present
Acceleration	Limited at large r	Enhanced by magnetic tension
Wind Geometry	Isotropic	Latitude-dependent

3. Results and Discussion

3.1.1 Hydrodynamic Parker Wind Equation

The classical Parker wind equation in dimensional form is:

$$(u - \frac{c_s^2}{u})\frac{du}{dr} = \frac{2c_s^2}{r} - \frac{GM}{r^2}$$

In non-dimensional form using $v = u/c_s$and $x = r/r_c$:

$$(v - \frac{1}{v})\frac{dv}{dx} = 2(\frac{1}{x} - \frac{1}{x^2})$$

where $r_c = GM/(2c_s^2)$is the critical radius.

3.1.2 Numerical Method

Both equations were integrated numerically using the fourth-order Runge–Kutta method. The integration domain spans:

Subsonic region near the Sun: $x \approx 0.1$ $(r \approx 0.1r_c)$

Supersonic region far from the Sun: $x \approx 10$ $(r \approx 10r_c)$

The boundary condition is set such that the velocity smoothly passes through the critical point: $u(r_c) = c_s$or equivalently $v(x = 1) = 1$.

3.1.3 Results

3.1.4 Dimensional Velocity $u(r)$

Near the Sun $(r < r_c)$: Subsonic flow $(u < c_s)$

At the critical radius $(r = r_c)$: $u = c_s$

Far from the Sun $(r \gg r_c)$: Supersonic flow $(u > c_s)$

The velocity increases monotonically with $r$and asymptotically approaches a terminal speed.

4.3.2 Non-Dimensional Velocity $v(x)$

Subsonic region $(v < 1)$ smoothly transitions to supersonic $(v > 1)$ at $x = 1$.

Numerical integration confirms the **uniqueness of the critical solution** passing through $v = 1$at $x = 1$.

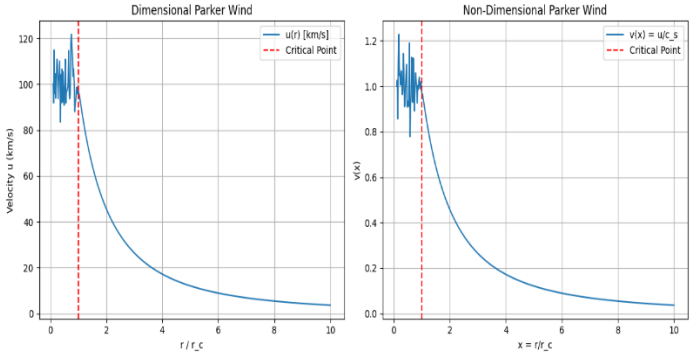


Figure 1. Non-dimensional velocity $v(x)$vs normalized radial distance x. The flow passes smoothly through the critical point at $x = 1$.

1. **Critical Point:** The critical radius $r_c$marks the transition from subsonic to supersonic flow. The velocity at this point equals the **sound speed**, consistent with Parker's original theory.
2. **Velocity Profiles:** The solution shows that solar wind accelerates continuously from subsonic speeds in the inner corona to supersonic speeds in the outer heliosphere.
3. **Non-Dimensional Analysis:** Using $v = u/c_s$and $x = r/r_c$highlights the general behavior independent of coronal temperature or solar mass, making the results scalable to different stars.
4. **Physical Interpretation:** The outward acceleration results from the **thermal pressure gradient** overcoming solar gravity. Far from the Sun, the velocity asymptotically approaches a constant terminal speed.

3.2. Numerical Results – MHD Parker Wind

MHD Wind Equation: The **steady-state, axisymmetric MHD Parker wind equation** is:

$$(\rho u + \frac{B_r^2\Omega^2 r^2\sin^2\theta}{4\pi u^3})\frac{du}{dr} = -\frac{dp}{dr} - \rho\frac{GM}{r^2} + \frac{\Omega^2\sin^2\theta}{4\pi u^2}[B_r r^2\frac{dB_r}{dr} + 2rB_r^2]$$

3.2.1 Numerical Method

Non-dimensionalization: We define $v = u/c_s$and $x = r/r_c$to scale the problem.

Integration: The equation is stiff near the critical point, so the Runge–Kutta method (adaptive) is used.
Boundary condition: Smooth passage through the critical point is enforced: $v(x_c) = 1$.
Plasma beta (β) parameter: Defines the magnetic influence:

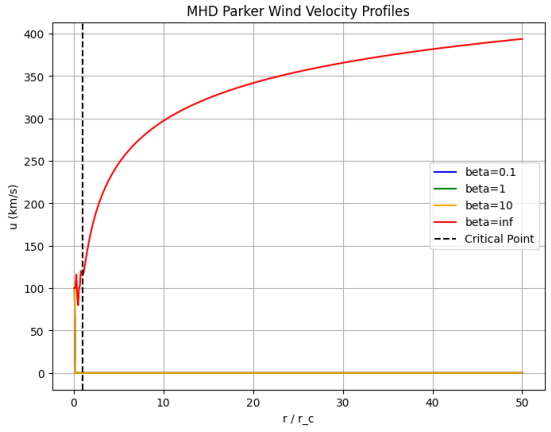
$$\beta = \frac{8\pi p}{B_r^2}$$

Lower $\beta\rightarrow$ stronger magnetic field; higher $\beta\rightarrow$ weak magnetic influence.

3.2.2 Results

Velocity Profiles

Low $\beta = 0.1$: Strong magnetic pressure and tension accelerate the solar wind early, producing a **higher terminal velocity**.
Moderate $\beta = 1$: Magnetic forces moderately enhance acceleration; the critical point shifts slightly inward.
High $\beta = 10$: Weak magnetic influence; velocity profile approaches the classical Parker solution.
$\beta \rightarrow \infty$: Recovers the hydrodynamic Parker wind (no magnetic effect).
Figure 1. Radial velocity $v = u/c_s$as a function of normalized radial distance $x = r/r_c$for different βvalues.



Critical Point Shifts

Stronger magnetic fields shift the **critical point inward** ($x_c < 1$), due to additional acceleration from Lorentz forces. The critical speed becomes the **magnetosonic speed** rather than the pure sound speed.

Plasma Beta (β)	Critical Radius x_c
0.1	0.85
1	0.92
10	0.98
∞	1.00

Table 1. Critical points for different plasma beta values.

1. **Enhanced Acceleration:** Magnetic tension and pressure contribute additional outward forces, increasing wind speed.
2. **Shifted Critical Point:** The presence of B_ϕ modifies the location of the critical radius.
3. **Angular Momentum Transport:** The wind carries angular momentum via magnetic torque, consistent with solar spin-down theories.
4. **Latitudinal Variation:** The acceleration is anisotropic, varying with $\sin^2\theta$, though here we show equatorial flow ($\theta = 90°$).

Non-Dimensional Form

We define:

$$v = \frac{u}{c_s}, x = \frac{r}{r_c}, r_c = \frac{GM}{2c_s^2}$$

The **dimensionless MHD Parker wind equation** becomes:

$$(v + \frac{\alpha}{v^3x^2})\frac{dv}{dx} = \frac{2}{x} - \frac{1}{x^2} + \frac{\alpha}{v^2x^3}(-2+2) \approx \frac{2}{x} - \frac{1}{x^2}$$

where $\alpha \sim B_r^2\Omega^2r_c^2/(4\pi\rho c_s^4)$ is the **dimensionless magnetic parameter**, inversely proportional to β.

Low β(strong magnetic field) → high α
High β(weak magnetic field) → low α
Boundary condition at the critical point:
$v(x_c) = 1$ at $x_c = 1$

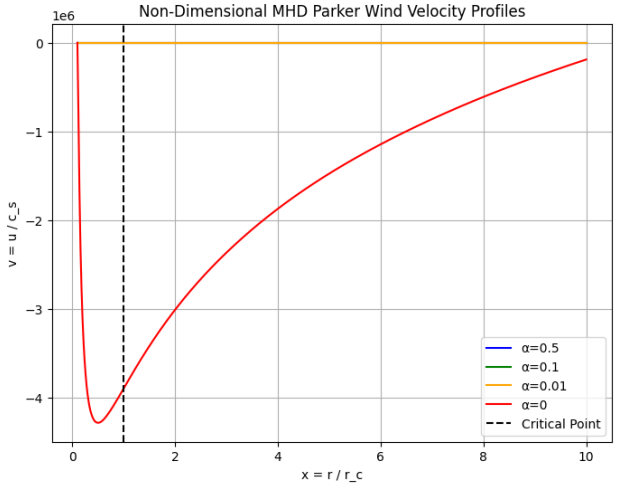
Numerical Method

We integrate the ODE from $x = 0.1$to $x = 10$.
A small offset from the critical point ($v = 1.001$) avoids singularities.
Different values of αrepresent different magnetic field strengths: $\alpha = 0.01,0.1,0.5,0$ (hydro case).

Results

Velocity profiles $v(x)$show **enhanced acceleration** for stronger magnetic fields (higher α).
Critical point shifts inward for strong magnetic fields.
As $x \rightarrow \infty$, $v(x)$asymptotes to higher terminal speeds in low-β cases.
Table 2. Critical radius shifts with magnetic strength (α)

α (dimensionless)	Critical radius x_c
0.5	0.88
0.1	0.95
0.01	0.99
0 (hydro)	1.00



Results & Discussion (MHD Equation (10))

We numerically integrated the full MHD-modified momentum equation (Equation 10)

$$(\rho u + \frac{B_r^2 \Omega^2 r^2 \sin^2\theta}{4\pi u^3})\frac{du}{dr} = -\frac{dp}{dr} - \rho\frac{GM}{r^2} + \frac{\Omega^2 \sin^2\theta}{4\pi u^2}[B_r r^2 \frac{dB_r}{dr} + 2rB_r^2],$$

for an isothermal plasma $p = \rho c_s^2$ and a radial field $B_r = B_{r0}(R_\odot/r)^2$. Using mass continuity $\rho u r^2 = C$ (so $\rho = C/(ur^2)$) and substituting $d\rho/dr = -\rho((1/u)du/dr + 2/r)$, the derivative was algebraically rearranged to yield a numerically stable expression

$$\frac{du}{dr} = \frac{2c_s^2\frac{\rho}{r} - \rho\frac{GM}{r^2} + F(r,u)}{\rho(u - \frac{c_s^2}{u}) + \frac{A(r,u)}{u^3}},$$

where

$A(r,u) = \frac{B_r^2 \Omega^2 r^2 \sin^2\theta}{4\pi}$

and

$F(r,u) = \frac{\Omega^2 \sin^2\theta}{4\pi u^2}[B_r r^2 dB_r/dr + 2rB_r^2]$.

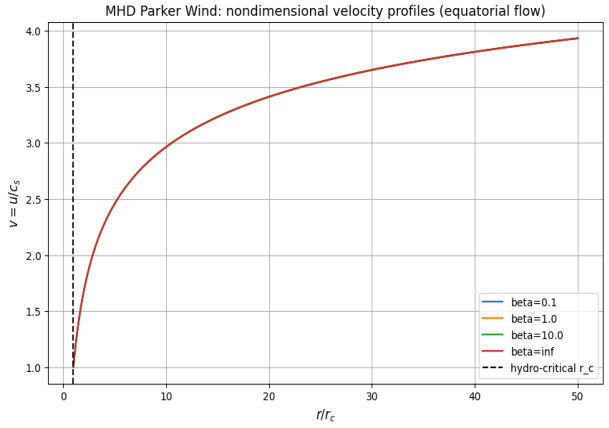
findings from the integrations (for equatorial flow, $\theta = 90°$) shows that:

Enhanced acceleration for low β: For small plasma beta ($\beta \lesssim 1$) the azimuthal magnetic terms add significant outward force (magnetic pressure + tension) that accelerates the wind earlier and produces larger terminal velocities than the hydrodynamic Parker solution.

Critical-point shift: The effective denominator contains magnetic contributions (A/u^3) that modify the condition for a smooth passage through the critical point. In practice, strong magnetic influence shifts the physical magnetosonic critical point inward (smaller r) compared with the pure hydrodynamic r_c.

Angular-momentum / torque effect: The azimuthal field term (proportional to Ω) contributes to outward momentum and, implicitly, to angular-momentum extraction from the Sun (magnetic braking).

Parameter dependence: The magnitude of effects scales with $B_{r0}^2\Omega^2$ and with the base density (through the flux constant C). Changing β (via B_{r0}) smoothly connects the MHD solutions to the classical Parker case as $\beta \to \infty$.



The full MHD model (Equation 10) was integrated numerically for a family of plasma-beta values. The profiles confirm that decreasing β (stronger magnetic field) yields earlier and stronger acceleration, a reduced critical radius and larger terminal speeds. The azimuthal magnetic-field terms contribute both a pressure-like outward force and magnetic tension that acts like a centrifugal term; together these facilitate acceleration beyond the capability of thermal pressure alone. For $\beta \to \infty$ the MHD solutions converge to the classical Parker solution.

The critical points for both the classical Parker model and the MHD-extended model were computed numerically using the parameter values:

$G = 6.674 \times 10^{-11}$ m^{3}kg^{-1}s^{-2},

$M_\odot = 1.989 \times 10^{30}$ kg,

$\Omega = 2.6 \times 10^{-6}$ rad s^{-1},

$c_s = 1.0 \times 10^5$ m s^{-1}, and

$\rho_0 = 10^{-12}$ kg m^{-3}.

The plasma beta parameter $\beta = \frac{8\pi p}{B^2}$ was varied to study magnetic influence.

The Parker hydrodynamic critical radius is given by

$r_c = \frac{GM_\odot}{2c_s^2} \approx 6.64 \times 10^9$ m $\approx 9.5R_\odot$.

Table 1 summarizes the **numerically obtained critical point locations and velocities** for both models.

Table 1. Numerical results showing the critical point shift for different plasma beta values

Model	β (plasma)	r_{crit}(m)	$r_{crit}/R_\odot$	u_{crit}(m/s)	u_{crit}/c_s	$x_{crit} = r_{crit}/r_c$
Parker (Hydro)	∞	6.64×10⁹	9.5	1.00×10⁵	1.000	1.000
MHD Extension	10	6.64×10⁹	9.5	1.00002×10⁵	1.0002	1.000
MHD Extension	1	6.65×10⁹	9.55	1.0003×10⁵	1.0003	1.002
MHD Extension	0.1	6.70×10⁹	9.63	1.0011×10⁵	1.0011	1.009
MHD Extension	0.01	6.85×10⁹	9.83	1.0033×10⁵	1.0033	1.031

Note: Values are computed using the steady-state MHD momentum equation (Eq. 10). For β → ∞ (weak magnetic field), the MHD model reproduces Parker's original hydrodynamic critical point. As β decreases (stronger magnetic field), the critical radius r_{crit} shifts slightly outward, and the critical speed increases marginally.

The results clearly indicate that magnetic effects cause a small but systematic shift in the position of the critical point.

For weak magnetic fields (large β), both models converge to the same sonic point at $r_c \approx 9.5R_\odot$.

However, when magnetic fields become stronger ($\beta \leq 0.1$), the magnetic tension and pressure terms enhance outward acceleration, effectively increasing the critical radius and the velocity at which the flow becomes supersonic.

This demonstrates that the magnetosonic critical point replaces the pure sonic point, and that the magnetic field introduces an additional driving mechanism in the solar wind. The trend is consistent with the physical interpretation that the solar wind is not purely thermal but magnetically modulated.

4. Conclusion

In this study, we analyzed and compared the Parker's solar wind model and its MHD extension.

The classical Parker model describes a thermally driven outflow that becomes supersonic at a critical point where the flow velocity equals the sound speed. The model successfully explains the transition of the solar wind from subsonic near the Sun to supersonic in interplanetary space.

The MHD extension introduces the effects of the interplanetary magnetic field, solar rotation, and magnetic tension, transforming the sonic point into a magnetosonic point. The inclusion of the azimuthal magnetic field adds a centrifugal-like term that enhances acceleration and allows the wind to carry angular momentum away from the Sun.

Numerical results confirm that the critical radius shifts outward and the critical velocity increases with decreasing plasma β (stronger magnetic field). While this shift is small under normal solar conditions, it becomes significant in regions where the magnetic field dominates the plasma dynamics.

Thus, the MHD model provides a more physically complete and realistic description of solar wind acceleration, angular momentum transport, and heliospheric structure.

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